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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Describes dynamic and static methods of obtaining data on projectile unbalance and procedures for computing dynamic and static unbalance. Applies to artillery projectiles.

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US ARMY TEST AND EVALUATION COMMAND
TEST OPERATIONS PROCEDURE

DRSTE-RP-702-103
*Test Operations Procedure 4-2-801   4 May 1978
AD No. A055596

PROJECTILE UNBALANCE

1. SCOPE. This TOP describes dynamic and static methods of obtaining
data on projectile unbalance and procedures for computing the static and
dynamic unbalance.

2. FACILITIES AND INSTRUMENTATION.

2.1 Facilities.

              ITEM   REQUIREMENTS

Shelter to provide protection   Concrete (or equivalent) floor,
from outdoor environments   level (less than 1.0% grade)

Crane or equivalent for   Capacity: 1/2-ton
lifting projectiles

Video camera   To remotely view the spinning
                                     operation of live projectiles

*This TOP supersedes TOP 4-2-801, 13 January 1972.

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2.2 Instrumentation.

| ITEM	MAXIMUM ERROR OF MEASUREMENT*
Dynamic balancing machine	±1.4 gr·cm per 45-kg weight of item (±0.05 oz·in per 100-lb weight of item)
Weight scales:	
Dynamic balancing**	±0.14 gr (±0.005 oz)
Static method	±0.03 gr (±0.001 oz)
Outside calipers**	±0.025 cm (±0.01 in)

* Values may be assumed to represent ±2 standard deviations; thus the stated tolerances should not be exceeded in more than 1 measurement out of 20.

** This instrumentation is required if the dynamic balancing machine requires calibration by initially spinning the projectile while placing known amounts of unbalance on the projectile.

3. PREPARATION FOR TEST.

3.1 Selection of Measurement Method. Unbalance in a projectile induces yaw and can seriously affect exterior ballistic performance. To enable the ballistician to correlate the unbalance of projectiles with range firings, the amounts of dynamic and static unbalance must first be determined. Both are usually prevalent to some degree in projectiles. Dynamic unbalance exists when the dynamic axis is misalined angularly with respect to the axis of rotation (α, fig 1); static unbalance exists when the center of gravity does not lie on the axis of rotation (ϵ).

For most practical purposes, unbalance in a rotating body whose axis of rotation is relatively short compared to its diameter is static unbalance which, in most cases, can be determined and corrected by the "level ways" method. A rotating body with a relatively long axis of rotation in proportion to its diameter (most projectiles are in this category) could also be statically balanced when checked on level ways but might be quite unbalanced when rotated; a balancing machine must therefore be used to determine the unbalance of such an object. When a projectile is balanced on a dynamic balancing machine, it will also be statically balanced.

$(\theta_a = 25^\circ)^*$

$(\lambda = 67^\circ)^*$

$(\theta_b = 118^\circ)^*$

$(\tau = 145^\circ)^*$
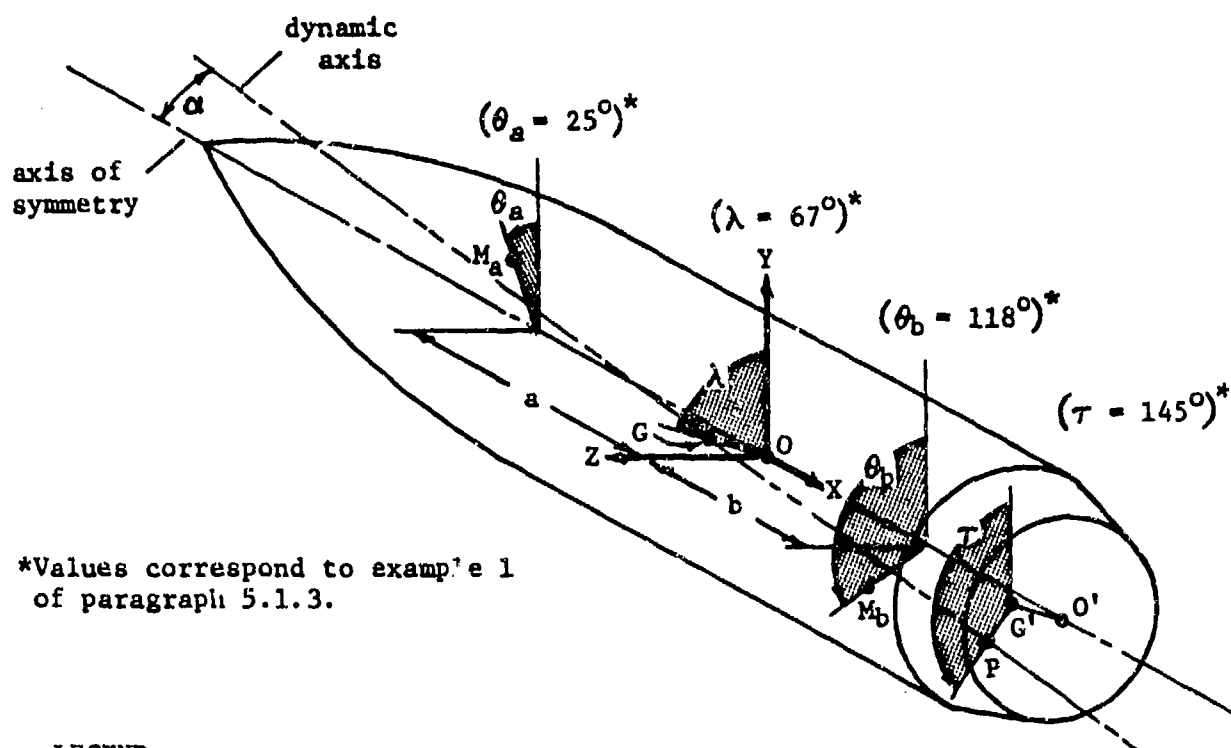
*Values correspond to example 1
of paragraph 5.1.3.

LEGEND:

α = angular difference (rad) between dynamic axis and axis of symmetry (rad)

O = center of gravity along axis of symmetry

G = point center of gravity in center of gravity plane

ϵ = OG = distance of point center of gravity (G) from axis of symmetry in center of gravity plane

OX, OY, OZ = mutually perpendicular coordinate axes with OY the reference from which angles are measured

a, b = axial distance from center of gravity to left correction plane and right correction planes respectively

M_a, M_b = unbalance in left and right correction planes respectively

θ_a, θ_b = angular location of unbalance in left and right correction planes respectively as measured from the reference (deg)

λ = angular position of OG relative to the reference

G' = projection of G in the base plane

P = point intersection of dynamic axis with base plane

O'G' = projection of ϵ in base plane

τ = angular position of P measured from the reference at point G' in base plane

Figure 1. Physical interpretation of projectile unbalance.

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3.2 <u>Projectiles</u>.

    a. Inspect the bearing surfaces of the projectiles to ensure a
smooth surface without nicks, built-up areas, or chipped paint.

    b. Mark the projectile (by a paint marker or tape) at a convenient
point to denote the reference from which all angles are to be measured.
For the static method of determining the unbalance, place three
additional marks 90° apart at the same axial location as the reference
mark. Label the four marks in the order shown in figure 4, paragraph
5.2. The angles between the four marks must not differ from 90° by
more than 4°.

    c. Determine the weight of each projectile and the axial location
of the center of gravity (measured from the base) in accordance with
TOP/MTP 4-2-800. For the dynamic method of determining unbalance, also
measure the transverse and axial moments of inertia.

3.3 <u>Dynamic Balancing Machine</u>. Ensure that the initial settings of the
electronic console, settings of optical devices, adjustment of bearings
and belts, and calibration are in accordance with the manufacturer's
instructions. For safety during tests of live projectiles, install a
video camera to remotely view the spinning operation.

3.4 <u>Data Requirements</u>. Record the following as applicable for each
projectile:

    a. Identification (type, model, etc.).

    b. Weight.

    c. Location of center of gravity (as measured from the base).

    d. Transverse (I_t) and axial (I_{ax}) moments of inertia.

4. <u>TEST CONTROLS</u>.

    a. When calibration of the dynamic balancing machine is accomplished
by spinning the projectile with a known unbalance, the same spin (rpm)
must be used during the performance test.

    b. The axial position of the projectile on the bearings and the
belt wrap around the projectile must remain the same throughout the
performance test.

    c. The sign convention for a and b in figure 1 must be used when
computing λ, ϵ, τ, and α by the equations in paragraph 5.1.3.

    d. For the static method, the axial location of the center of
gravity must coincide with the center of the length of the V-block.

5. PERFORMANCE TESTS.

> NOTE: Data are recorded in US customary units in the following
>   tests since these units are used by most equipment
>   currently in use. When SI units apply, substitute
>   appropriate units and conversion factors.

5.1 Determining Unbalance by Dynamic Method.

5.1.1 Method.

a. Select two peripheral surfaces that are smooth and concentric
with the projectile axis of symmetry for spin bearing surfaces.

b. Place the projectile on the balancing machine (fig 2) in
accordance with the manufacturer's instructions.

c. Record the location of the left and right correction planes
with respect to the center of gravity (a and b, fig 1).

d. Operate the balancing machine in accordance with the
instruction manual. Increase the spin of the projectile until the
spin rpm is the same as used for calibration (or within the manufacturer's
specifications for the balancing machine). Two test runs are required
to determine projectile unbalance parameters: one for the right
correction plane and one for the left correction plane.

5.1.2 Data Required. Record the following for each projectile using
the appropriate data collection sheet in appendix A:

a. Left and right correction plane distances from projectile
center of gravity (a and b of figure 1 respectively).

b. Amounts of unbalance in left and right correction planes
(M_a and M_b of fig 1 respectively).

c. Angular locations of unbalance in left and right correction
planes (θ_a and θ_b of fig 1 respectively).
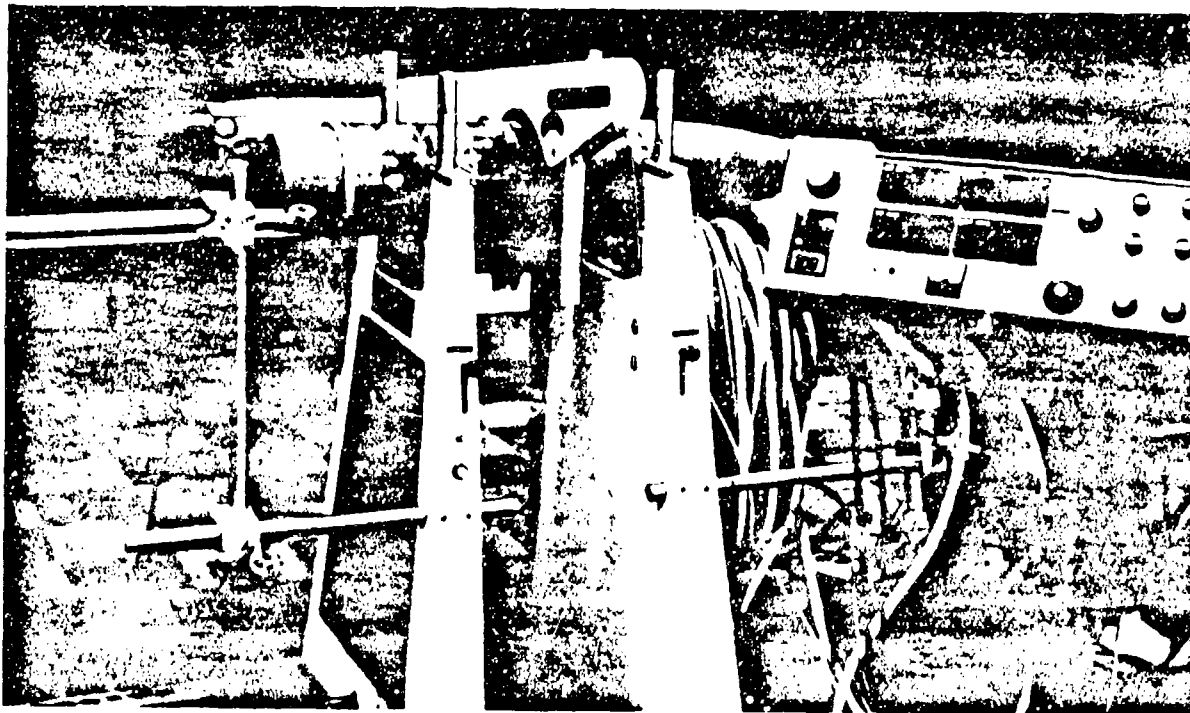
5
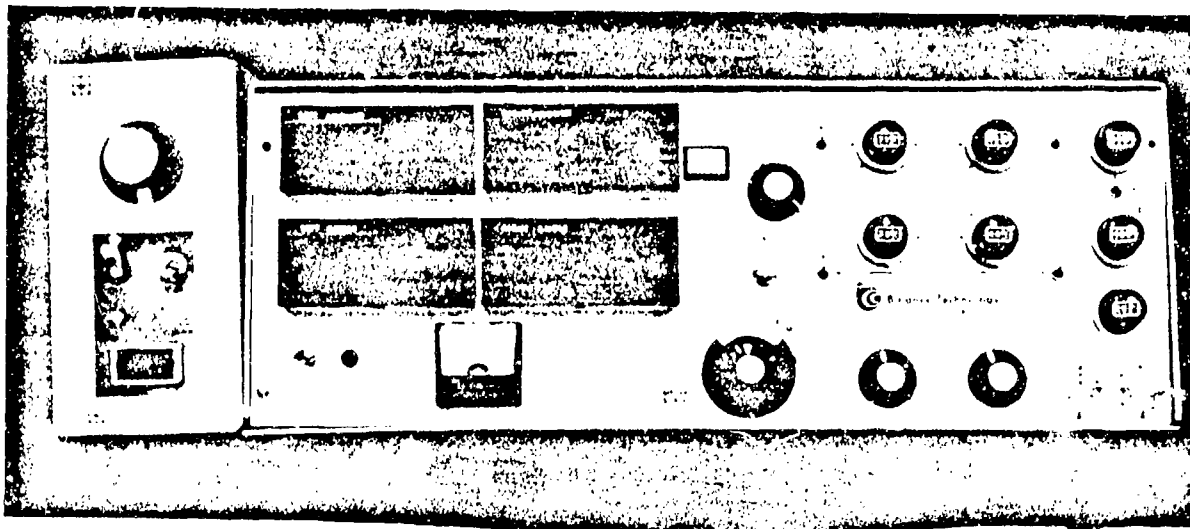
Figure 2.   Typical dynamic balancing machine.

Figure 3.   Console (for above machine).

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5.1.3 <u>Unbalance Computations</u>. Compute the static and dynamic unbalance (λ, ϵ, τ, and α, fig 1) as shown by the examples below. A suggested method of evaluating the $\tan^{-1}$ function is given in appendix B. The angles λ and τ are measured in the same direction as θ_a and θ_b and from the same reference when standing at the base of the projectile. In comparing λ and τ from balancing data from a balancing machine that measures the angles in opposite directions, the numerical values for λ and τ will not be the same but will be related by

$$\lambda \text{(cw)} = 360° - \lambda \text{(ccw)}$$

$$\tau \text{(cw)} = 360° - \tau \text{(ccw)}$$

<u>Example 1</u>

Given the following unbalance data

  a = -2.2 in. (this is a minus value because of its direction)

  b = 5.0 in.

  M_a = 4.6 oz·in. $\theta_a = 25°$ ⎫
                                              ⎬ measured ccw standing at the base
  M_b = 3.9 oz·in. $\theta_b = 118°$ ⎭

For a projectile weighing 102.9 lb and having the following moments of inertia

  I_t = 40.240 lb·ft^2 (transverse)

  I_{ax} = 3.750 lb.ft^2 (axial)

Static unbalance is determined by

$$\epsilon = \frac{1}{16W} (M_a{}^2 + M_b{}^2 + 2M_aM_b \cos \theta)^{1/2} \text{ (in)}$$

$$\lambda = \tan^{-1} \left(\frac{M_b \sin \theta}{M_a + M_b \cos \theta} \right) + \theta_a \text{ (deg)}$$

where

  W = weight of projectile (lb)

  $\theta = \theta_b - \theta_a$

Substituting given values

$$\epsilon = \frac{[(4.6)^2 + (3.9)^2 + 2(4.6)(3.9) \cos (93)]^{1/2}}{16(102.9)}$$

$\epsilon = 3.6$ mi's $= 3.6 \times 10^{-3}$ in.

$$\lambda = \tan^{-1} \left[\frac{3.9 \sin (93)}{4.6 + 3.9 \cos (93)} \right] + 25°$$

$$\lambda = \tan^{-1} \left(\frac{3.89}{4.40} \right) + 25° = 42° + 25° = 67°$$

$\lambda \sim 67°$

Dynamic unbalance is determined by

$$\alpha = \frac{1}{2304C} \left[(aM_a)^2 + (bM_b)^2 + 2abM_aM_b \cos \theta \right]^{1/2} \text{ (rad)}$$

$$\tau = \tan^{-1} \left(\frac{bM_b \sin \theta}{aM_a + bM_b \cos \theta} \right) + \theta_a \text{ (deg)}$$

where

$C = I_t - I_{ax}$ (lb·ft^2)

$\theta = \theta_b - \theta_a$

Substituting given values

$$\alpha = \frac{[(-2.2 \times 4.6)^2 + (5.0 \times 3.9)^2 + 2(-2.2)(5.0)(4.6)(3.9) \cos (93)]^{1/2}}{2304 (40.240 - 3.750)}$$

$\alpha = 2.67 \times 10^{-4}$ rad $= 2.67 \times 10^{-4} \times \frac{180°}{\pi}$

$\alpha = 0.0152°$

$$\tau = \tan^{-1} \left[\frac{(5.0)(3.9) \sin (93)}{(-2.2)(4.6) + (5.0)(3.9) \cos (93)} \right] + 25°$$

$$\tau = \tan^{-1} \left(\frac{19.47}{-11.14} \right) + 25° = -60° + 180° + 25°$$

$\tau = 145°$ (see fig 1) measured ccw when standing at the base.

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Example 2

Given the following unbalance data

  a = 2.2 in.

  b = 5.0 in.

  M_a = 3.2 oz·in. θ_a = 155°

  M_b = 6.9 oz·in. θ_b = 280°

For a projectile weighing 102.9 lb and having the following moments of inertia

  I_t = 40.240 lb·ft^2 (transverse)

  I_a = 3.750 lb·ft^2 (axial)

Static unbalance is determined by

$$\epsilon = \frac{1}{16W} (M_a^2 + M_b^2 + 2M_aM_b \cos \theta)^{1/2} \text{ (in)}$$

$$\lambda = \tan^{-1} \left(\frac{M_b \sin \theta}{M_a + M_b \cos \theta} \right) + \theta_a \text{ (deg)}$$

where

  W = weight of projectile (lb)
  $\theta = \theta_b - \theta_a$

Substituting given values

$$\epsilon = \frac{[(3.2)^2 + (6.9)^2 + 2(3.2)(6.9) \cos (125)]^{1/2}}{16(102.9)}$$

$$\epsilon = 3.5 \text{ mils} = 3.5 \times 10^{-3} \text{ in.}$$

$$\lambda = \tan^{-1} \left[\frac{6.9 \sin (125)}{3.2 + 6.9 \cos (125)} \right] + 155°$$

$$\lambda = \tan^{-1} \left(\frac{5.65}{-0.76} \right) + 155° = -82° + 180° + 155° \text{ (app B)}$$

$$\lambda = 253°$$

Dynamic unbalance is determined by

$$\alpha = \frac{1}{2304C}\left[(aM_a)^2 + (bM_b)^2 + 2abM_aM_b\cos\theta\right]^{1/2} \text{ (rad)}$$

$$\tau = \tan^{-1}\left(\frac{bM_b\sin\theta}{aM_a + bM_b\cos\theta}\right) + \theta_a \text{ (deg)}$$

where

$$C = I_t - I_{ax} \text{ (lb·ft}^2)$$

$$\theta = \theta_b - \theta_a$$

Substituting given values

$$\alpha = \frac{[(-2.2 \times 3.2)^2 + (5.0 \times 6.9)^2 + 2(-2.2)(3.2)(5.0)(6.9)\cos(125)]^{1/2}}{2304(40.240 - 3.750)}$$

$$\alpha = 4.6 \times 10^{-4} \text{ rad} = 4.6 \times 10^{-4} \times \frac{180°}{\pi}$$

$$\alpha = 0.027°$$

$$\tau = \tan^{-1}\left[\frac{(5.0)(6.9)\sin(125)}{(-2.2)(3.2) + (5.0)(6.9)\cos(125)}\right] + 155°$$

$$\tau = \tan^{-1}\left(\frac{28.26}{-26.83}\right) + 155° = -46° + 180° + 155° \text{ (app B)}$$

$$\tau = 289°$$

5.2 Determining Unbalance by Static Method.

5.2.1 Method.

a. Measure and record the distance between the fulcrum and the support on the scale (H, fig 4) and the angle of the V-block (U).

b. Place the projectile on the V-block so that the axial location of the center of gravity coincides with the center of the V-block length.

c. Adjust the counterweight so that no more than 5% of the total weight of the projectile is on the weighing-scale pan.

d. Rotate the projectile to aline the 0° reference mark with the reference mark on the V-block. Record data after all motion (due to handling of the projectile) has ceased.

e. Rotate the projectile to aline the 90°, 180°, and 270° marks with the mark on the V-block and record data as when alining with the 0° mark (d above).
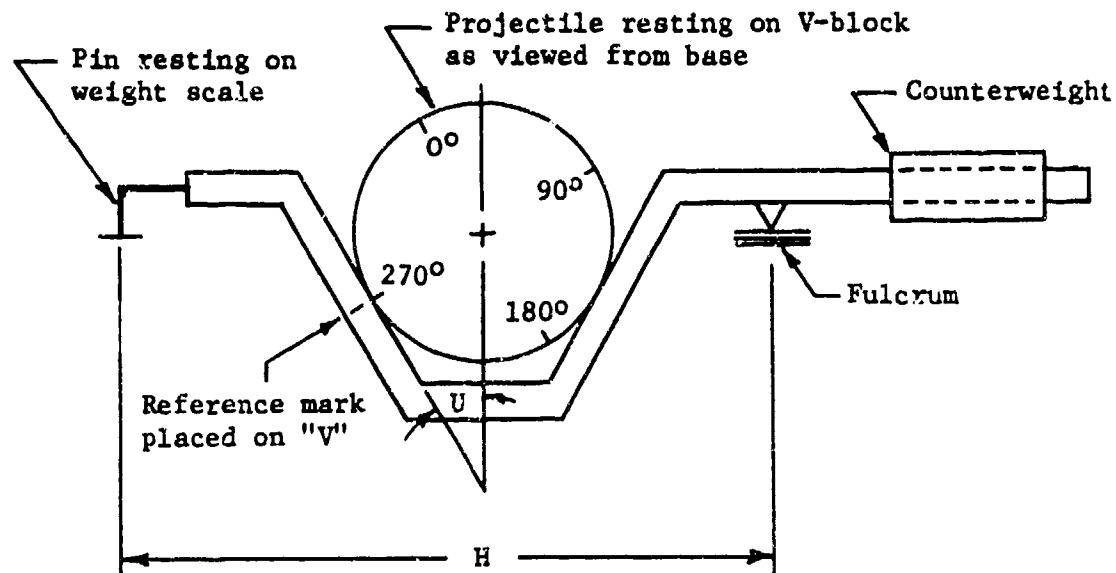
Figure 4. Equipment for measuring static unbalance.

5.2.2 Data Required. Record the following data for each projectile using the appropriate data collection sheet in appendix A:

a. Values of H (in.) and U (degrees) for the equipment used.

b. Weights as read from the weight scale for each of the four positions.

5.2.3 Unbalance Computation. Compute static unbalance by the following equation:

$$\epsilon = \frac{H}{2W} \left[(F_1 - F_3)^2 + (F_2 - F_4)^2 \right]^{1/2} \text{ (in)}$$

$$\lambda = U + \tan^{-1} \left(\frac{F_2 - F_4}{F_1 - F_3} \right) \text{ (deg)}$$

where

   H = distance from fulcrum to support on scale

   U = angle of V-block

   F_1 = weight scale reading corresponding to 0° position (lb)

   F_2 = weight scale reading corresponding to 90° position (lb)

   F_3 = weight scale reading corresponding to 180° position (lb)

   F_4 = weight scale reading corresponding to 270° position (lb)

An example is given below. The angle λ is to be measured in a clockwise direction when standing at the base of the projectile.

Example

Given the following unbalance data

F_1 = 2.949   H = 6 in.

F_2 = 2.937   W = 102.9 lb

F_3 = 2.851   U = 30°

F_4 = 2.864

Substituting given values

$$\epsilon = \frac{6[(2.949 - 2.851)^2 + (2.937 - 2.864)^2]^{1/2}}{(2)(102.9)}$$

$$\epsilon = 0.0036$$

$$\lambda = 30° + \tan^{-1}\left(\frac{2.937 - 2.864}{2.949 - 2.851}\right)$$

$$\lambda = 30° + \tan^{-1}\left(\frac{0.0730}{0.0980}\right) = 30° + 37° = 67°$$

     The line to be marked on the projectile to denote λ would be marked from the 0° reference measuring 67° cw when standing at the base of the projectile.

6.   DATA REDUCTION AND PRESENTATION.

    a.   Tabulate all data.

    b.   Use these data to compute the effect of unbalance on initial yaw and jump. 1/ 2/

> Recommended changes to this publication should be forwarded to
> Commander, U. S. Army Test and Evaluation Command, ATTN:
> DRSTE-AD-M, Aberdeen Proving Ground, MD 21005. Technical infor-
> mation may be obtained from the preparing activity: Commander,
> U. S. Army Aberdeen Proving Ground, ATTN: STEAP-MT-M, Aberdeen
> Proving Ground, MD 21005. Additional copies are available from
> the Defense Documentation Center, Cameron Station, Alexandria,
> VA 22314. This document is identified by the accession number
> (AD No.) printed on the first page.

1/ Sterne, T. E., "On Jump Due to Bore Clearance," Aberdeen Proving
   Ground, MD, USABRL Report 491, September 1944.
2/ Hitchcock, H. P., "Examples of Jump of Shell Due to Yaw," Aberdeen
   Proving Ground, MD, USABRL Memo Report 899, June 1955.

APPENDIX A

DATA COLLECTION SHEETS

DATA SHEET FOR DYNAMIC METHOD

Date _____

Project _____

Engineer _____

Sheet _____ of _____

Projectile ID	Projectile Weight	Moments of Inertia		Left Plane			Right Plane		
		Traverse (I_t)	Axial (I_{ax})	Distance from CG (a)	Unbalance (M_a)	Angle (θ_a)	Distance from CG (b)	Unbalance (M_b)	Angle (θ_b)

DATA SHEET FOR STATIC METHOD

Date _____   Sheet _____ of _____

Project _____

Engineer _____

Projectile ID	Projectile Weight	V-Block Data		Weight Scale Readings			
		H	U	0° Position	90° Position	180° Position	270° Position

APPENDIX B
EVALUATING THE TAN^{-1} FUNCTION FOR λ AND T

The following is a suggested method for evaluating the tan^{-1} function for λ and T when the signs of the numerator and denominator are known. The angles λ and T are to be measured in the same direction as θ_a, θ_b.

Sign of Numerator (N)	Sign of Denominator (D)	Evaluation of tan^{-1} (N/D) by Example*
+	+	tan^{-1} (1.73/1) = 60°
+	−	tan^{-1} (1.73/−1) = − 60° + 180° = 120°
−	+	tan^{-1} (−1.73/1) = − 60° = 300°**
−	−	tan^{-1} (−1.73/−1) = + 60° + 180° = 240°

* In this procedure tan (N/D) is treated as an add function [i.e., tan (−1.73) = −tan (1.73)] and 180 is only added to tan^{-1} (θ) when D < 0.

** A negative angle in this application is converted to a positive angle by adding 360°; i.e., − 60° + 360° = 300°.

STEAP-MT-M

SUBJECT: Review of TOP 4-2-801, Projectile Unbalance

CDR, USA Yuma Proving Ground, ATTN: STEYP-MMI, Yuma, AZ 85364
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CDR, Tooele Army Depot, ATTN: SDSTE-MAE (Mr. Joe Carter, Mr. Ken Arvey),
Tooele, UT 84075

1. Reference letter, HQ, TECOM (DRSTE-ME), 10 October 1977, subject:
Guide for the Preparation of Test Operations Procedures (TOPs).

2. Request that inclosed TOP 4-2-801 be reviewed for technical adequacy
and that concurrence or any recommended changes be forwarded to reach
this office by 5 April 1978. Any changes recommended should be explained
and drafted exactly as desired to ensure precise accommodation.

FOR THE COMMANDER:

1 Incl
as

JOHN A. FEROLI
Chief, Methodology and
Instrumentation Division
Materiel Testing Directorate

Incl 2